AC Power Theory From Poynting Theorem: Accurate Identification of Instantaneous Power Components in Nonlinear-Switched Circuits

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Abstract—**This paper contributes to narrowing the long-standing theoretical gap with power theory (or "power definitions") for nonlinear ac switching circuits. The true instantaneous energy transformation and storage components of ac circuits are identified from the Poynting Theorem. This paper tackles the problem of power identification from the most general form of energy conservation. Therefore, it is no longer necessary to mathematically "define" powers to fit the engineering solution of a problem. The identification technique does not present problems with physical meaning since it is in full agreement with Maxwell's Equations. In this paper, the method is applied to the identification of the power components of single-phase switched circuits. Instantaneous energy is decomposed only into energy transformed (related to active power) and energy stored (related to reactive power). Examples that have caused physical interpretation problems with other power theories are presented for illustration and validation.**

Index Terms—**Active power, alternating current circuits, energy restored, energy stored, energy transformed, instantaneous power, nonlinear circuits, power definitions, power theory, reactive power.**

I. Introduction

POWER definitions have been the subject of much research for more than 100 years. At the end of the 19th century, Steinmetz generated and compiled most of the available knowledge for the analysis of power in ac circuits in [1]. The problems with the definition of power (and power factor) for unbalanced circuits were identified as early as 1920 [2]; there are more than 70 pages with discussions on the definition of power. A second round of discussions took place in 1933 [3] and focused on the definition of reactive power for nonlinear circuits. Companion papers and discussions extend to almost 60 pages. The discussions on what power really is in nonlinear and unbalanced circuits have continued and a great number of papers have been published on the matter. We have compiled more than 200 papers on the subject. In 2000, the IEEE published the standard 1429-2000 on power definitions that has served to further ignite the discussions [4].

One can learn about the issues and history of power definitions through some of the papers of Emanuel [5]–[9] and Czarnecki [10]–[16] and their references.

Numerous attempts to establish a power theory that fits some observable phenomena have produced a gamut of power concepts lacking physical meaning when applied to other cases. Through mathematical manipulations, many authors have tried to generalize power definitions applicable to a particular case. Some authors, for example, Shepherd and Fang [17], have explicitly admitted the weaknesses of their power theories in regards to physical interpretation.

In this paper, we propose an instantaneous power theory directly derived from Maxwell's Equations and specifically from the Poynting Vector Theorem. Accordingly, only two energy (or power) components exist: 1) the energy transformed yielding the active power and 2) the energy that is stored/restored in the electromagnetic fields that gives birth to the reactive power. Note that the power components of this paper are not defined from a particular example, but are accurately computed from the most fundamental conservation of energy principles. Only the instantaneous information on terminal voltage and current are required to fully characterize the power phenomena of a switching load.

There are publications in favor of [18] and against [19] the use of the Poynting vector to describe power phenomena in electrical circuits. For us, the Poynting vector is not merely a mathematical tool for calculating energy flow as claimed in [19]. Poynting theorem has been derived from the experimentally macroscopically undisputed Maxwell's Equations. Therefore, it offers the best physical description yet available for the representation of electrical power and energy phenomena.

This paper advocates for the time-domain analysis of powers for nonlinear circuits. Note that the commonly used quantities to characterize power such as apparent power S, reactive power Q, power factor PF, etc., do not exist in instantaneous terms. Those quantities are simply definitions that have shown to be useful and fully meaningful only for linear ac circuits [20]. Extensions to nonlinear circuits have failed to provide the same physical meaning.

The contribution of this paper is the proper identification of the instantaneous energy (and power) components for nonlinear-switched ac circuits. The results are different from the instantaneous power theories of Fryze [21] and Akagi *et al.* [22]. Both of those power theories lack sound physical meaning

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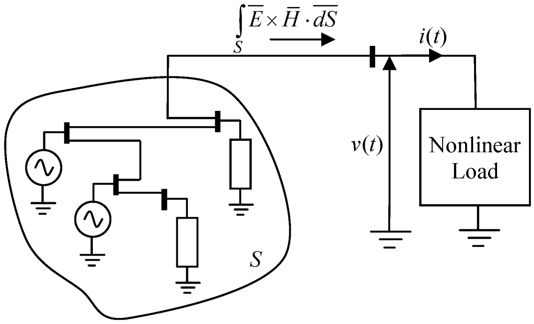

Fig. 1. Application of the Poynting theorem to a source and a nonlinear load.

even for very simple circuits. For example, both theories produce a reactive power component for cases without energy storage; see, for example, [23] and [24]. Note that some researchers, relying on classical definitions, believe that reactive power can occur without changes in the energy stored [16]. However, in physics, power is the rate of change of energy. When energy does not change, power must be null. Therefore, we state that any definition of power that cannot be related to changes in energy must be recognized as physically incorrect, even if it is useful for engineering purposes.

This paper focuses on nonlinearities in single-phase switched circuits. Future research will look into other types of nonlinearities and unbalanced multiphase circuits.

II. Physical Instantaneous Power Theory

In order to be completely general, the flow of power should be analyzed from Maxwell's Equations and, in particular, by means of the Poynting Vector Theorem (PVT) published in 1884 [25]. Fig. 1 shows the application realm of the method: a nonlinear load is fed from a (complex) source. The resulting power quantities have full physical meaning and are applicable to all conditions. The most general equation describing the transfer of power between a source and a load is

$$\oint_S E\times H\cdot dS = -\int_V E\cdot J\, dV - \frac{\partial}{\partial t}\int_V \left(\frac{H\cdot B}{2}+\frac{E\cdot D}{2}\right) dV. \tag{1}$$

The term on the left-hand side is interpreted as the total power transferred. The first term on the right-hand side represents the power that is transformed into other forms of energy, given by Joule's Law, and the second term corresponds to the time variations of the energy stored in the electric and magnetic fields. Equation (1), when applied to electrical circuits, can be written as shown in (2) at the bottom of the page.

The instantaneous power transferred from the source to the load is given by $p(t)$ and computed from the product $v(t)i(t)$. The power transformed, represented by the active power $a(t)$, must follow Joule's Law with fidelity. Changes in the energy stored must be properly characterized by the instantaneous reactive power $r(t)$ in [W]. This instantaneous reactive power can be inductive and/or capacitive, thus $r(t) = r_L(t)+r_C(t)$. Equation (2) is completely general; it applies equally to linear and nonlinear circuits. In (2), it is also implicit that all other powers commonly defined for the analysis of nonlinear circuits, such as fictitious, distortion, scattered, nonactive, etc., have no electromagnetic existence. Only active and reactive powers exist in electromagnetic (Maxwell's Equations) sense.

The term "instantaneous reactive power" is used in this paper to refer to the rate of change of the instantaneous energy stored. Since energy can only be stored/restored to/from the so-called "reactive" elements (inductors and capacitors), rather than inventing a new term or awkwardly carrying throughout "rate of change of energy stored," we have preferred to be consistent with the common practice for ac circuits. Note that our instantaneous reactive power $r(t)$ is different from the Akagi's instantaneous reactive power.

Two cases are analyzed in this paper: 1) when all of the elements of the circuit are known, and the more general and yet unresolved case 2) when only the voltage and current at the load terminals, which is seen as a black box, are known. The selected examples are periodic ac single-phase circuits; however, the method is completely general and can be applied even during the transient state.

A. Instantaneous Power for Known Circuit Elements

The fundamental relationships between current, voltage, power, and energy for the basic circuit elements are known. An ideal resistor must dissipate (and not store) power according to Joule's Law (Ri^2). When the resistance and the current through a resistor are known, the instantaneous active power is

$$a(t) = Ri_R(t)^2. \tag{3}$$

An ideal inductor stores/restores energy in its magnetic field according to Maxwell as $(1/2)Li^2$. The instantaneous power of an inductor must be all reactive power and must be computed from the time derivative of the instantaneous energy as

$$r_L(t) = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}Li_L(t)^2\right]. \tag{4}$$

$$\begin{aligned}\text{power transferred}(t) &= \text{power transformed}(t) + \frac{\mathrm{d}}{\mathrm{dt}}[\text{energy stored}(t)]\\ p(t) &= v(t)i(t) = \text{ active power}(t) + \text{reactive power}(t)\\ p(t) &= a(t)+r(t).\end{aligned} \tag{2}$$

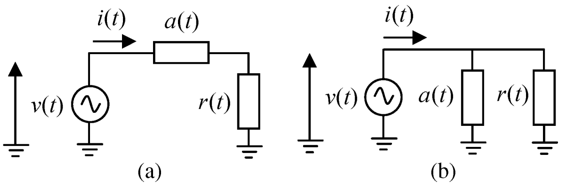

Fig. 2. Models for the representation of power phenomena in nonlinear circuits. (a) Series equivalent circuit. (b) Parallel equivalent circuit.

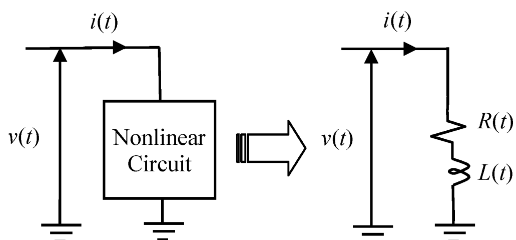

Fig. 3. Time-varying equivalent circuit with identical energy dissipation and storage characteristics.

Similarly, a capacitor stores/restores energy in its electric field as $(1/2)Cv^2$. Therefore, the instantaneous reactive power of a capacitor should be computed from

$$r_C(t) = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}Cv_C(t)^2\right]. \tag{5}$$

B. Identification of Power Components Only From the Terminal Voltage and Current Measurements

In practice, the circuit elements of a load are not known. The load consists of a complex arrangement of circuit elements and switches. The general problem consists in identifying the instantaneous active and reactive powers from the knowledge of terminal voltage and current only. The load is seen as a black box (Fig. 1).

To identify the active and reactive components of power, one needs to start by establishing a model for the circuit. Two circuit models are possible for the Poynting theorem: series [26] and parallel [21]; see Fig. 2. We have selected the series arrangement because for ac circuits, the excitation is the voltage. In addition, for most practical purposes, the voltage can be considered to be a perfect sinusoidal function [27]. Therefore, all of the information about the particularities of the load can only be obtained from the instantaneous current, which is the response of the circuit. The current waveshape contains all of the information on the energy exchanges between a source and a load, as confirmed in (10).

A sensible model for switched circuit loads is to assume that the energy is dissipated by constant resistors and that energy is stored/restored by constant inductors. Coherent with realistic switched loads (motors, lamps, power supplies, etc.), our model does not include a capacitor, but it can be included when needed (see Section III-G). Fig. 3 describes the underlying identification principle of this paper. We compute an equivalent time-varying circuit comprised of a time-varying resistor in series with a time-varying inductor. The instantaneous energy dissipated and stored/restored in the equivalent circuit is identical instantaneously to that of the unknown nonlinear circuit.

From (3) and (4), we obtain the instantaneous power for a series linear R–Lcircuit as

$$p(t) = R(t)i(t)^2 + \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}L(t)i(t)^2\right]. \tag{6}$$

From basic physics, we know that power is the time derivative of energy. Consequently, there is no power (consumed or stored/restored) if energy does not change with time. Therefore, the instantaneous energy can be computed from the integral of the instantaneous power as

$$w(t) = \int p(t)\mathrm{dt} = \int R(t)i(t)^2\mathrm{dt} + \int \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}L(t)i(t)^2\right]\mathrm{dt}. \tag{7}$$

In the case of a switched electrical circuit, the resistance and inductance remain constant between switching operations. Thus, R and L can be taken out of the integral and derivatives, yielding

$$p(t) = R_k i(t)^2 + L_k i(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t) \tag{8}$$

and

$$w(t) = R_k \int i(t)^2\mathrm{dt} + L_k \int i(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t)\mathrm{dt}. \tag{9}$$

We have added a subindex k to R and L to indicate that they are constant between switching operations, but are allowed to have a finite number of values in the study period. Equations (8) and (9) form a set of linearly independent equations, where R_k and L_k are the unknowns, given by

$$\begin{bmatrix} i(t)^2 & i(t)\frac{d}{dt}i(t) \\ \int i(t)^2 dt & \int i(t)\frac{d}{dt}i(t)dt \end{bmatrix}\begin{bmatrix} R_k \\ L_k \end{bmatrix} = \begin{bmatrix} p(t) \\ w(t) \end{bmatrix}. \tag{10}$$

A proof that (8) and (9) are linearly independent functions is given in the Appendix. Equation (9) can be seen mathematically as a moment of (8). However, this paper is centered in physics rather than in math.

Once R_k and L_k are computed from (10), one can use (3) and (4) to obtain the instantaneous power dissipation and power storage components. These components are consistent with physics since they are extracted from the power and energy equations only. This is a clear distinction with all available methods that create new power definitions to serve engineering purposes. See, for example, [4], where a collection of powers is defined mathematically and a physical meaning is sometimes not found [8].

Note that the matrix in (10) is only a function of the instantaneous current $i(t)$, its time derivatives, and integrals. The voltage enters, multiplied by the current, only in the right-hand term given by $p(t)$ and $w(t)$. This indicates that all of the information of the energy phenomena in a circuit can be obtained from the characteristics of the instantaneous current.

For the numerical solution of (10), one must rely on, easy to obtain, digital measurements of the instantaneous voltage $v(t)$ and current $i(t)$ at the load terminals. Assuming that current and

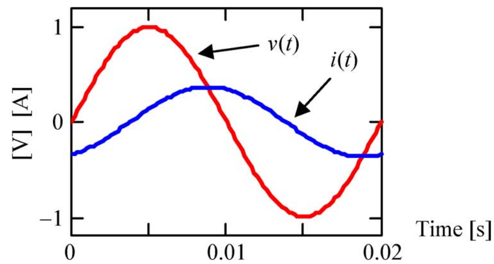

Fig. 4. Instantaneous current and voltage for a linear R–L circuit.

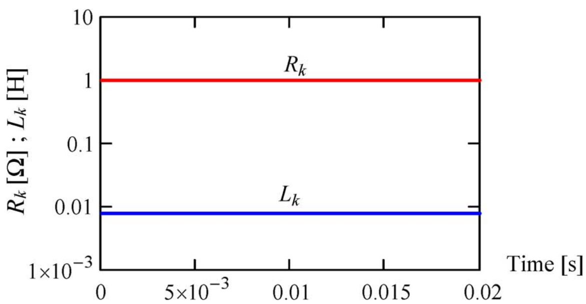

Fig. 5. Calculated results for the solution of (11) for an R–L circuit.

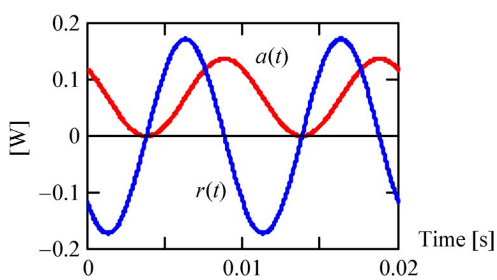

Fig. 6. Instantaneous active and reactive powers corresponding to the signals of Fig. 4.

voltage are measured with a constant sampling rate Δt, (10) becomes

$$\begin{bmatrix} i(t_k)^2 & i(t_k)\frac{\mathrm{d}}{\mathrm{dt}}i(t)|_{t=t_k} \\ \int\limits_{t_k}^{t_k+\Delta t} i(t)^2\mathrm{dt} & \int\limits_{t_k}^{t_k+\Delta t} i(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t)\mathrm{dt} \end{bmatrix} \begin{bmatrix} R_k \\ L_k \end{bmatrix} = \begin{bmatrix} p(t_k) \\ w(t_k) \end{bmatrix}. \tag{11}$$

The discrete power and energy functions (right-hand side) are computed from

$$p(t_k) = v(t_k)i(t_k) \tag{12}$$

and

$$w(t_k) = \int\limits_{t_k}^{t_k+\Delta t} p(t)\mathrm{dt} = \int\limits_{t_k}^{t_k+\Delta t} v(t)i(t)\mathrm{dt}. \tag{13}$$

All of the elements in (11) and the solution for R_k and L_k can be conveniently obtained numerically. In this paper, we have used commercially available computing packages (MathCAD and Matlab) for all numerical calculations using built-in functions. Note that one needs to solve the matrix (11) for each sample t_k. Thus, we identify a pair of R_k and L_k values for each sample, and physics takes care of the variations.

III. Illustration Examples

Several examples are used to illustrate the application and validity of the method. Equation (11) is solved for each discretized (in time) point of voltage $v(t)$ and current $i(t)$ measurement at the load terminals.

A. Linear R-L Circuit

This first example is intended to explain the calculation method. Consider that the measured current and voltage at the terminals of a linear ac circuit are given in Fig. 4. The measurements correspond to a series linear R–L circuit ($R = 1\ \Omega$; $L = 8$ mH) with

$$v(t) = \sin(\omega t); \quad i(t) = \frac{1}{Z}\sin(\omega t - \phi) \tag{14}$$

where

$$\omega = 2\pi f = 2\pi(50) = 100\pi$$
$$Z = \sqrt{R^2 + (\omega L)^2} = 2.705\ \Omega$$
$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right) = 68.303016°.$$

We have solved the system of (11) for every t_k. Fig. 5 shows the results for the entire study time. One can appreciate that the method perfectly identifies the values of the resistive and inductive components. Once R_k and L_k are known, we use (3) and (4) to compute the instantaneous power consumed and rate of change of the stored energy; see Fig. 6. One can see that $a(t)$ and $r(t)$ are double frequency functions in accordance with physics. $a(t)$ is always greater than zero, demonstrating that energy can only be consumed (and not returned) to the source from a passive circuit. $r(t)$ is a symmetric sinusoid wave with zero average, showing that all stored energy in the inductor is restored to the source.

The condition number of the matrix in (11) as a function of time is plotted in Fig. 7. For a sampling rate of $\Delta t = 200\ \mu$s (100 points per cycle), the maximum occurs at $t_k = 3.6$ ms with a value of 1.58×10^8. Although the matrix is ill-conditioned in regions using a sufficient number of significant digits for the calculations, it is possible to obtain accurate results. The corresponding equation for the worst case at $t_k = 3.6$ ms is

$$\begin{bmatrix} 5.103\times10^{-4} & -2.617 \\ 3.312\times10^{-8} & -2.548\times10^{-4} \end{bmatrix}\begin{bmatrix} R_k \\ L_k \end{bmatrix} = \begin{bmatrix} -0.020439 \\ -2.0064\times10^{-6} \end{bmatrix}. \tag{15}$$

Experimentation varying the sampling rate showed that the condition number increases as the sampling rate increases. The maximum condition number for $\Delta t = 100\ \mu$s (200 points per cycle) is 6.76×10^8. For $\Delta t = 40\ \mu$s (500 points per cycle), the maximum condition number becomes 5.4×10^9. For $\Delta t = 20\ \mu$s (1000 points per cycle), the maximum condition number is 5.71×10^{10}. Accurate identification results were always obtained for all sampling rates.

B. Controlled Rectifier Feeding a Resistive Load

The case of a controlled rectifier has been used to discredit several of the available power theories [22]. Some available the-

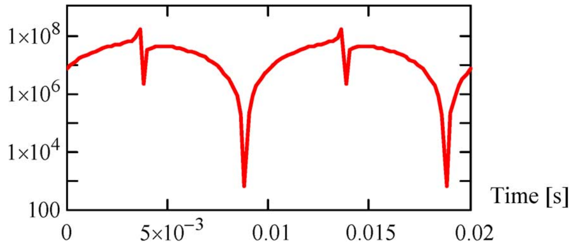

Fig. 7. Variation of the condition number with time for the family of matrices corresponding to Example A: Linear R– L circuit (signals of Fig. 4).

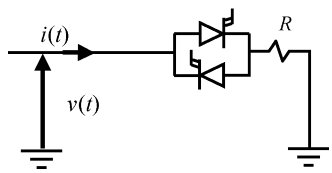

Fig. 8. Controlled rectifier feeding a resistive load.

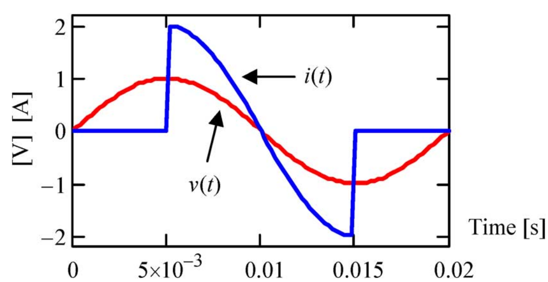

Fig. 9. Current and voltage for a controlled rectifier feeding a resistive load.

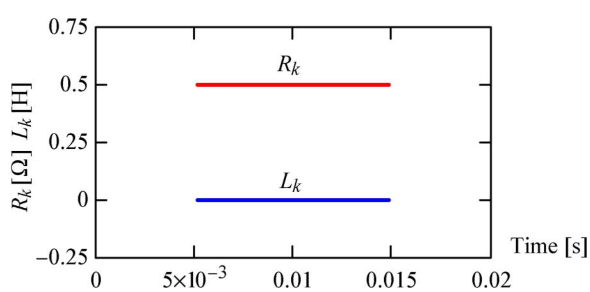

Fig. 10. Calculated results for the solution on (11) for a controlled rectifier feeding a resistive load.

ories predict the existence of reactive power in a circuit incapable of storing energy [24]. Consider the circuit of Fig. 8, with $R = 0.5\,\Omega$, where the firing times (or angles) have been adjusted to give the current shown in Fig. 9. The results of the identification process are shown in Fig. 10. One can appreciate that the identification is visibly correct. The computed inductance is in the order of 10^{-12} due to roundoff error.

The method properly computes the correct power dissipated $a(t) = Ri(t)^2$ with (3); see Fig. 11. Following the laws of physics, $p(t)$ is equal to $a(t)$ over the entire period. This means that all power supplied by the source is consumed by the load and there is no reactive power when there are no elements in the circuit capable of storing energy. This is also corroborated

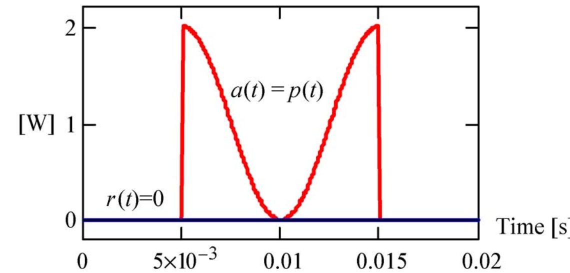

Fig. 11. Instantaneous active and reactive powers corresponding to the voltage and current signals of Fig. 9.

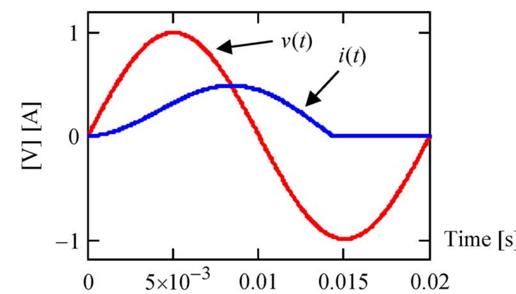

Fig. 12. Voltage and current for a half-wave rectifier feeding an R–L load.

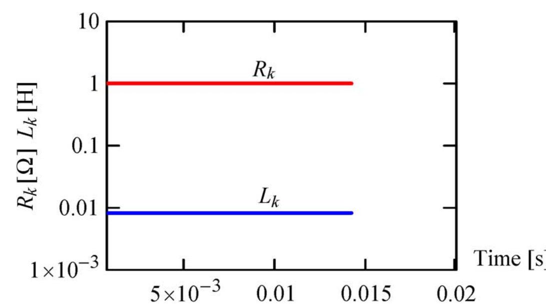

Fig. 13. Calculated results for the solution on equation (11) for a half-wave rectifier feeding an R–L load.

with the fact that $r(t) = 0$ when computed with (4) since $L_k \approx 0$ (10^{-12} H). For the periods where the current is zero, reflecting reality, (11) does not exist and, thus, there is no possible solution.

C. Half-Wave Rectifier Feeding an R-L Load

Fig. 12 shows the input voltage and current drawn by a half-wave rectifier feeding an R–L series load; where $R = 1\ \Omega$ and $L = 0.8$ mH. The results of the identification procedure of (11) are shown in Fig. 13. One can see that the method computes fairly accurately the correct values for R_k and L_k. A spike caused by the abrupt chopping of the current at zero crossing at the end of the conduction time has been filtered out. Spikes present no implementation complications because they can be easily identified numerically since the derivative becomes very large at discontinuity points. Fig. 14 shows the active and reactive components of power computed from (3) and (4). As before, the results are in full agreement with physics: $a(t) \geq 0$ and the net area under $r(t)$, its integral over a period, is zero.

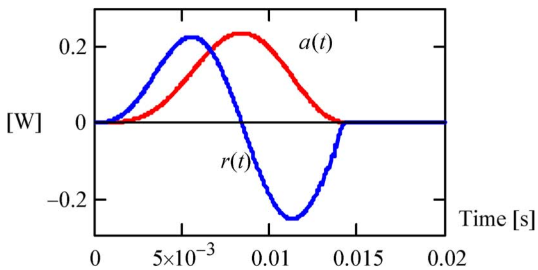

Fig. 14. Instantaneous active and reactive powers corresponding to the voltage and current signals of Fig. 12.

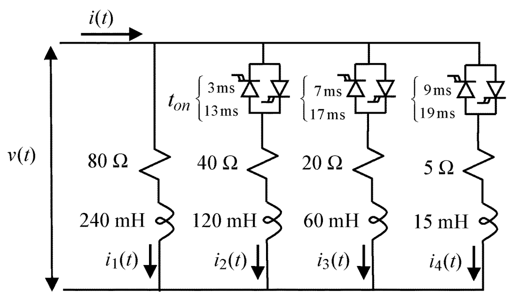

Fig. 15. Switching R–L branches with the same time constant.

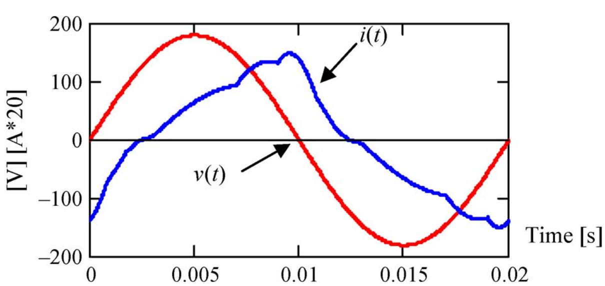

Fig. 16. Terminal voltage and current for the circuit of Fig. 15.

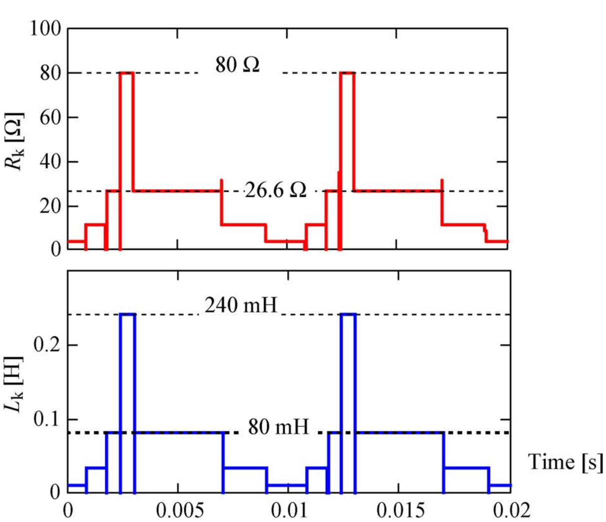

Fig. 17. Results of the identification process for the circuit of Fig. 15.

D. Several Parallel Controlled Rectifiers Feeding R-L Loads With the Same Time Constant

The next example consists of a circuit with four R–L parallel-switched branches (with different switching times) as illustrated in Fig. 15. Note that although the resistors and inductors have different values, the time constant of all the branches is the same ($L/R = 3$ ms). Fig. 16 shows the applied voltage and the resulting current obtained with time-domain simulations to reach steady state. There are 1000 points per cycle in the simulation. We have used the terminal voltage and the (computed) terminal current in (11)–(13) to identify the circuit elements. The results of the identification technique are shown in Fig. 17. The resistance and inductance computed with the method are equal to the equivalents when reducing the parallel circuits, given by

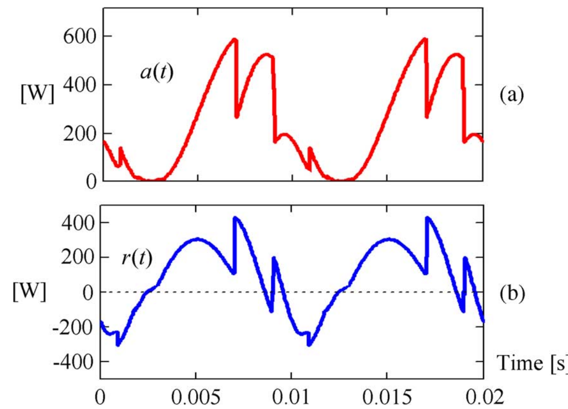

Fig. 18. Instantaneous active $a(t)$ and reactive $r(t)$ powers for the circuit of Fig. 15. (a) Active power. (b) Reactive power.

$$
\begin{aligned}
R_{\mathrm{eq}1} &= 80\ \Omega; \quad L_{\mathrm{eq}1} = 240\ \mathrm{mH}\\
R_{\mathrm{eq}2} &= \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = 26.66\ \Omega; \quad L_{\mathrm{eq}2} = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}} = 80\ \mathrm{mH}\\
R_{\mathrm{eq}3} &= \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = 11.43\ \Omega;\\
L_{\mathrm{eq}3} &= \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}} = 34.29\ \mathrm{mH}\\
R_{\mathrm{eq}4} &= 3.48\ \Omega; \quad L_{\mathrm{eq}4} = 10.43\ \mathrm{mH}.
\end{aligned}
$$

Once $R(t)$ and $L(t)$ are known, the instantaneous active and reactive powers $a(t)$ and $r(t)$ can be easily obtained from (3) and (4) (see Fig. 18). One can appreciate that in previous examples and in perfect agreement with physics, the instantaneous reactive power $a(t)$ is always positive (or zero) and the instantaneous reactive power $r(t)$ has a zero average.

E. General Case—Two Parallel R-L Loads With a Different Time Constant

The next example consists of a circuit with two R–L branches with a different time constant as shown in Fig. 19. The current and voltage shapes are given in Fig. 20. The results of the identification are presented in Fig. 21. When the switched branch is off, the method properly identifies the values of the fixed branch. For the transient region, when both branches are on, our method obtains time-varying (not constant as before) equivalents $R(t)$ and $L(t)$. This is consistent with theoretical expectations because it is not possible to obtain a constant parameter equivalent circuit for an R–L-switched circuit when the time constants are different [28]. During a transient, there is no completely accurate methodology of combining two or more parallel circuits

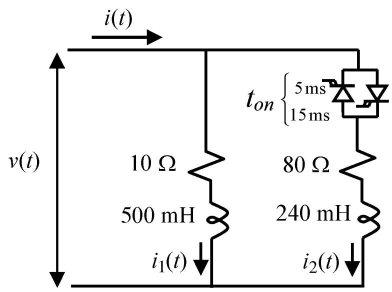

Fig. 19. Two parallel branches with different time constants.

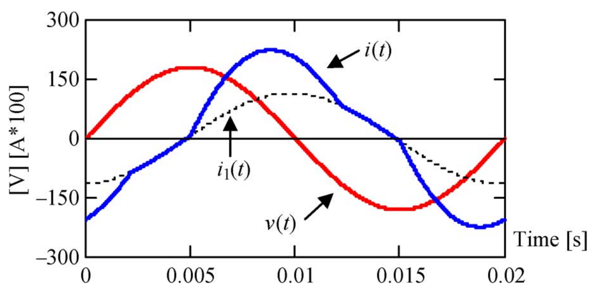

Fig. 20. Terminal voltage and current for the circuit of Fig. 19.

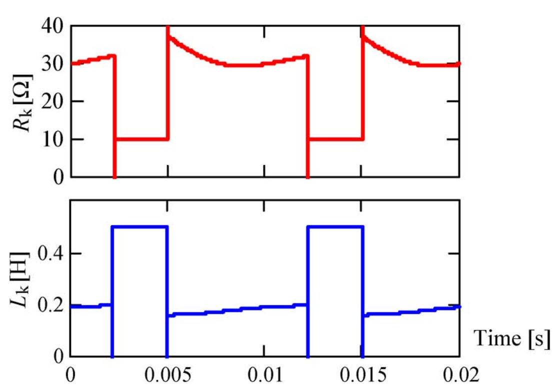

Fig. 21. Results of the identification process for the circuit of Fig. 19.

with different L/R into a single circuit with a constant value of L/R. Fig. 22 shows the instantaneous reactive and active power components. A similar physical interpretation as with previous examples can be given. $a(t)$ shows consumption with no energy returned to the source, while $r(t)$ has zero average, indicating that all energy that is stored in the inductors is restored to the source.

F. Nonsinusoidal Voltage Excitation

The voltage in a power system is, for most practical cases, sinusoidal [27]. However, to demonstrate the generality of method proposed in this paper, a case with a large third voltage harmonic component is presented. Consider a series R–L circuit ($R = 2\ \Omega$; $L = 5$ mH) fed by the following voltage:

$$v(t) = \sin(\omega t) + \frac{1}{2}\sin(3\omega t). \tag{16}$$

Fig. 23 shows the voltage and current waveshapes. The results of the identification are presented in Fig. 24. One can see that the identification of R and L is very good. The instantaneous active and reactive power components are shown in Fig. 25. As

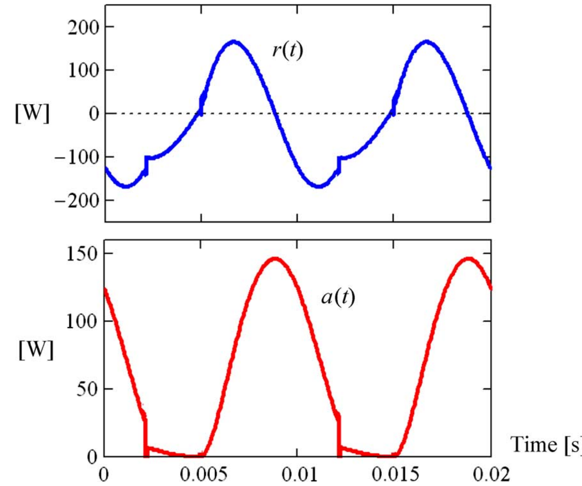

Fig. 22. Instantaneous reactive $r(t)$ and active $a(t)$ power components for the circuit of Fig. 19.

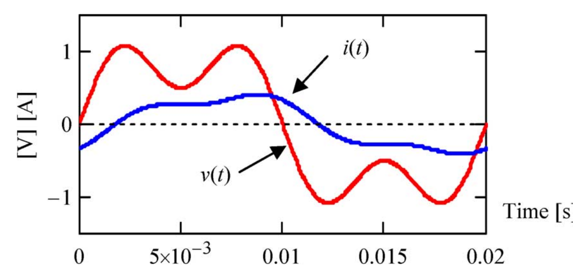

Fig. 23. Nonsinusoidal voltage excitation and corresponding current.

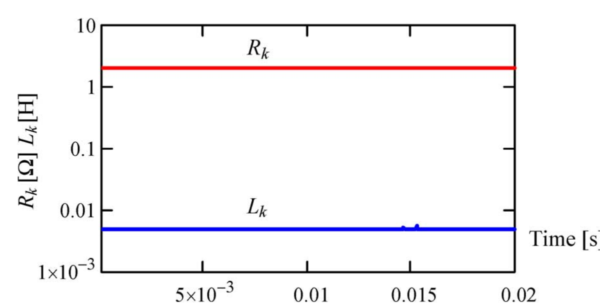

Fig. 24. Results of the identification process applied to the signals of Fig. 23.

before, $a(t)$, the instantaneous power consumed, is always positive or zero, while $r(t)$, the instantaneous reactive power, has zero average.

G. Capacitive Load

The last example consists of a linear R–C series load ($R = 1\ \Omega$; $C = 4$ mF). The terminal voltage and current are (see Fig. 26)

$$v(t) = \sin(\omega t); \quad i(t) = \frac{1}{Z}\sin(\omega t + \phi) \tag{17}$$

where

$$\omega = 2\pi f = 2\pi(50) = 100\pi$$
$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = 1.278\ \Omega$$
$$\varphi = \tan^{-1}\left(\frac{1}{\omega CR}\right) = 38.512^\circ.$$

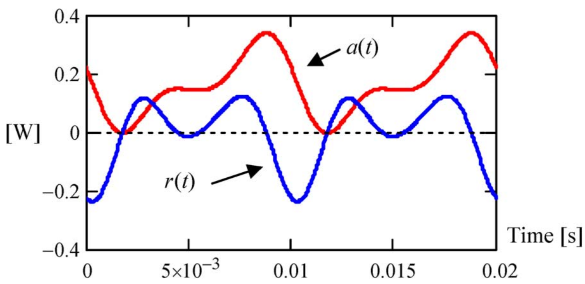

Fig. 25. Instantaneous active and reactive powers for the signals of Fig. 23.

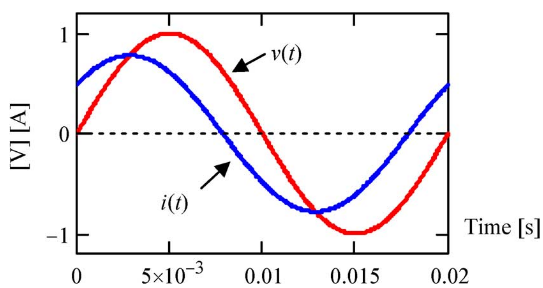

Fig. 26. Terminal voltage and current for the capacitive circuit.

Since most loads in a power system are inductive, the identification technique proposed in this paper was written for inductive loads. However, the underlying theory (Poynting Theorem) is completely general and can be applied to any load. The best indication of the presence of a capacitive load is when the computed inductance is negative. This is because for a given current, the energy stored instantaneously in an inductor is in the opposite direction as the instantaneous energy stored in a capacitor. The equivalent capacitor can be computed by combining (4) and (5) as

$$r_L(t) = i(t)L(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t) = r_C(t) = i(t)\left(\frac{1}{C(t)}\int i(t)\mathrm{dt}\right) \tag{18}$$

yielding

$$C(t) = \frac{1}{L(t)}\frac{\int i(t)\mathrm{dt}}{\frac{\mathrm{d}}{\mathrm{dt}}i(t)}. \tag{19}$$

For linear cases, (19) reduces to $C(t) = -1/[\omega^2 L(t)]$. Fig. 27 shows the results of the application of (11)–(13) to the voltage and current shapes illustrated in Fig. 26. One can see that R_k is identified directly and that the corresponding L_k is negative ($= -2.533$ mH). Using (19), we can obtain the proper numerical value for the capacitor C_k.

In Fig. 29, the instantaneous active and reactive powers are presented. The active power component $a(t)$ is obtained from (3) as $R(t)i(t)^2$. The reactive power $r(t)$ can be computed from (18) or using (4) and (5) as follows (they are all equivalent):

$$r(t) = \frac{1}{2}L(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t)^2 \tag{20a}$$

$$r(t) = \frac{1}{2}C(t)\frac{\mathrm{d}}{\mathrm{dt}}\left[v_C(t)\right]^2$$

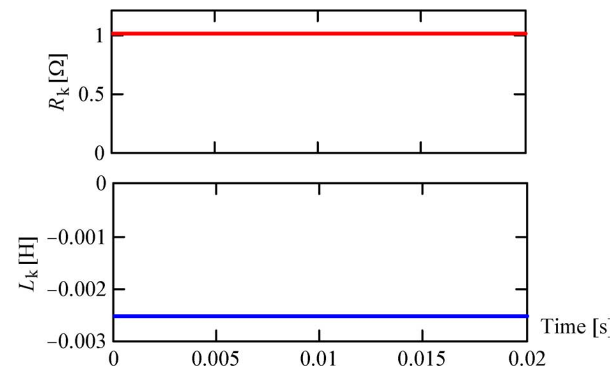

Fig. 27. Results of the identification process for the waveshapes of Fig. 26.

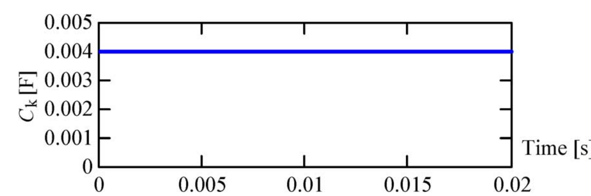

Fig. 28. Results of the identification process for the waveshapes of Fig. 26.

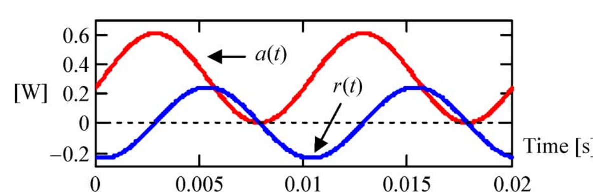

Fig. 29. Results of the identification process for the waveshapes of Fig. 26.

$$r(t) = \frac{1}{2}C(t)\frac{\mathrm{d}}{\mathrm{dt}}\left[v(t) - R(t)i(t)\right]^2. \tag{20b}$$

IV. Conclusion

This paper contributes to narrowing the long-standing theoretical gap with power theory (or "power definitions") for nonlinear circuits. Based only on terminal measurements of instantaneous voltage and current, the true (in the Maxwell sense) energy transformation and storage components of ac circuits have been identified from the Poynting Vector Theorem.

In this paper, the method has been applied successfully to the identification of a large number of switched circuits. Instantaneous energy is decomposed only into energy transformed (related to active power) and energy stored (related to "reactive" power), thereby eliminating any physical interpretation issues.

The system identification method, using a function and its integral, has applications beyond the identification of elements of ac circuits.

Appendix
Linear Independence of (8) and (9)

Let $f_1(t)$ and $f_2(t)$ be functions defined over an interval I as

$$f_1(t) = p(t) = Ri^2(t) + Li(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t) \tag{21}$$

$$f_2(t) = w(t) = R\int i(t)^2\mathrm{dt} + L\int i(t)\frac{\mathrm{d}}{\mathrm{dt}}i(t)\mathrm{dt}. \tag{22}$$

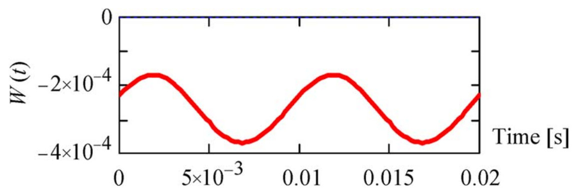

Fig. 30. Wronskian of (8) and (9) for the linear circuit R–L of Example A.

The instantaneous power and energy $p(t)$ and $w(t)$ of electrical circuits are differentiable functions in the intervals between switching. If two functions are linearly dependent for every t in the interval, their Wronskian

$$W(t) = \begin{vmatrix} f_1(t) & f_2(t) \\ f_1'(t) & f_2'(t) \end{vmatrix} \equiv 0. \tag{23}$$

Consequently, if for some t in the interval I, the Wronskian has a value $W(t) \neq 0$, then the functions $f_1(t)$ and $f_2(t)$ are linearly independent [29].

In ac electrical circuits, the instantaneous current $i(t)$ is a periodic function and its Fourier series exists. Therefore, it suffices to show that for $i(t) = Im \sin(\omega t + \varphi)$, the Wronskian is different than zero for some t to prove that (8) and (9) are linearly independent. Fig. 30 shows the variation of the Wronskian $W(t)$ with respect to time for the R–L circuit of Example1 ($\mathrm{R} = 1\ \Omega$; $L = 8$ mH). There are 100 samples in Fig. 30. We can see that the Wronskian is different than zero at every t in the period. Therefore, functions (8) and (9) are linearly independent at every t.

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